7.4 Tower Design

Once the external loads acting on the tower are determined, one proceeds with an analysis of the forces in various members with a view to fixing up their sizes. Since axial force is the only force for a truss element, the member has to be designed for either compression or tension. When there are multiple load conditions, certain members may be subjected to both compressive and tensile forces under different loading conditions. Reversal of loads may also induce alternate nature of forces; hence these members are to be designed for both compression and tension. The total force acting on any individual member under the normal condition and also under the broken- wire condition is multiplied by the corresponding factor of safety, and it is ensured that the values are within the permissible ultimate strength of the particular steel used.

Bracing systems

Once the width of the tower at the top and also the level at which the batter should start are determined, the next step is to select the system of bracings. The following bracing systems are usually adopted for transmission line towers.

Single web system (Figure 7.29a)

It comprises either diagonals and struts or all diagonals. This system is particularly used for narrow-based towers, in cross-arm girders and for portal type of towers. Except for 66 kV single circuit towers, this system has little application for wide-based towers at higher voltages.

Double web or Warren system (Figure 7.29b)

This system is made up of diagonal cross bracings. Shear is equally distributed between the two diagonals, one in compression and the other in tension. Both the diagonals are designed for compression and tension in order to permit reversal of externally applied shear. The diagonal braces are connected at their cross points. Since the shear perface is carried by two members and critical length is approximately half that of a corresponding single web system. This system is used for both large and small towers and can be economically adopted throughout the shaft except in the lower one or two panels, where diamond or portal system of bracings is more suitable.

Pratt system (Figure 7.29c)

This system also contains diagonal cross bracings and, in addition, it has horizontal struts. These struts are subjected to compression and the shear is taken entirely by one diagonal in tension, the other diagonal acting like a redundant member.

It is often economical to use the Pratt bracings for the bottom two or three panels and Warren bracings for the rest of the tower.

Portal system (Figure 7.29d)

The diagonals are necessarily designed for both tension and compression and, therefore, this arrangement provides more stiffness than the Pratt system. The advantage of this system is that the horizontal struts are supported at mid length by the diagonals.

Like the Pratt system, this arrangement is also used for the bottom two or three panels in conjuction with the Warren system for the other panels. It is specially useful for heavy river-crossing towers.

Where

p = longitudinal spacing (stagger), that is, the distance between two successive holes in the line of holes under consideration,

g = transverse spacing (gauge), that is, the distance between the same two consecutive holes as for p, and

d = diameter of holes.

For holes in opposite legs of angles, the value of 'g' should be the sum of the gauges from the back of the angle less the thickness of the angle.

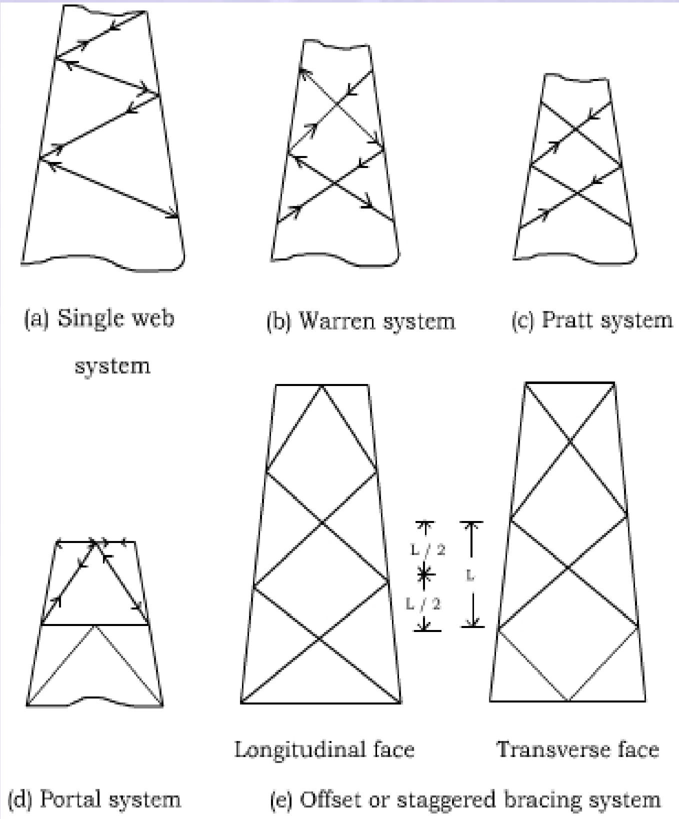


Figure 7.29 Bracing syatems

Net effective area for angle sections in tension

In the case of single angles in tension connected by one leg only, the net effective section of the angle is taken as

$$A_{eff} = A + Bk \qquad \textbf{(7.28)}$$

Where

A = net sectional area of the connected leg,

B = area of the outstanding leg = (l -t)t,

l = length of the outstanding leg,

t = thickness of the leg, and

$$k = \frac{1}{1 + 0.35\frac{B}{A}}$$

In the case of a pair of angles back to back in tension connected by only one leg of each angle to the same side of the gusset,

$$k = \frac{1}{1 + 0.2\frac{B}{A}}$$

The slenderness ratio of a member carrying axial tension is limited to 375.

7.4.1 Compression members

While in tension members, the strains and displacements of stressed material are small, in members subjected to compression, there may develop relatively large deformations perpendicular to the centre line, under certain criticallol1ding conditions.

The lateral deflection of a long column when subjected to direct load is known as buckling. A long column subjected to a small load is in a state of stable equilibrium. If it is displaced slightly by lateral forces, it regains its original position on the removal of the force. When the axial load P on the column reaches a certain critical value P_{cr}, the column is in a state of neutral equilibrium. When it is displaced slightly from its original position, it remains in the displaced position. If the force P exceeds the critical load P_{cr}, the column reaches an unstable equilibrium. Under these circum- stances, the column either fails or undergoes large lateral deflections.

Table 7.30 Effective slenderness ratios for members with different end restraint

Type of member	KL / r
a) Leg sections or joint members bolted at connections in both faces.	L/r
b) Members with eccentric loading at both ends of the unsupported panel with value of L / r up to and including 120	L/r
c) Members with eccentric loading at one end and normal eccentricities at the other end of unsupported panel with values of L/r up to and including 120	30+0.75 L/r
d) Members with normal framing eccentricities at both ends of the unsupported panel for values of L/r up to and including 120	60+0.5 L/r
e) Members unrestrained against rotation at both end of the unsupported panel for values of L/r from 120 to 200.	L/r
f) Members partially restrained against rotation at one end of the unsupported panel for values of L/r over 120 but up to and including 225	28.6+0.762 L/r
g) members partially restrained against rotation at both ends of unsupported panel for values of L/r over 120 up to and including 250	46.2+0.615 L/r

Slenderness ratio

In long columns, the effect of bending should be considered while designing. The resistance of any member to bending is governed by its flexural rigidity EI where I =Ar2. Every structural member will have two principal moments of inertia, maximum and minimum. The strut will buckle in the direction governed by the minimum moment of inertia. Thus,

$$I_{min} = Ar_{min}^2 \qquad \textbf{(7.29)}$$

Where r_{min} is the least radius of gyration. The ratio of effective length of member to the appropriate radius of gyration is known as the slenderness ratio. Normally, in the design procedure, the slenderness ratios for the truss elements are limited to a maximum value.

IS: 802 (Part 1)-1977 specifies the following limiting values of the slenderness ratio for the design of transmission towers:

Leg members and main members in the cross-arm in compression	150
Members carrying computed stresses	200
Redundant members and those carrying nominal stresses	250
Tension members	350

Effective length

The effective length of the member is governed by the fixity condition at the two ends.

The effective length is defined as 'KL' where L is the length from centre to centre of intersection at each end of the member, with reference to given axis, and K is a non-dimensional factor which accounts for different fixity conditions at the ends, and hence may be called the restraint factor. The effective slenderness ratio KL/r of any unbraced segment of the member of length L is given in Table 7.30, which is in accordance with 18:802 (Part 1)-1977.

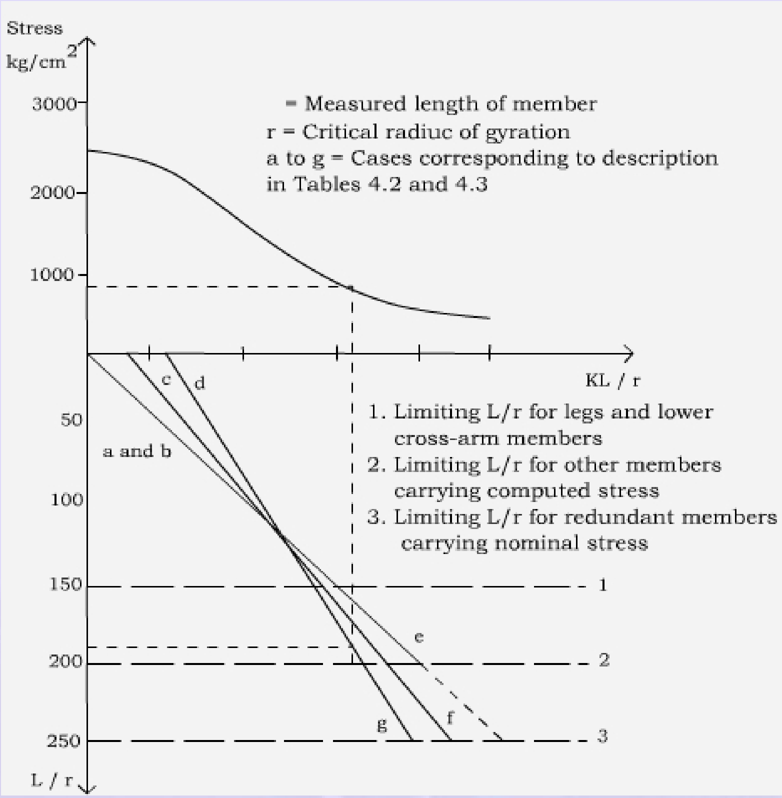


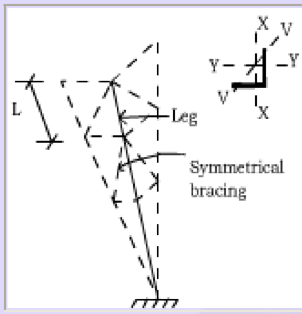
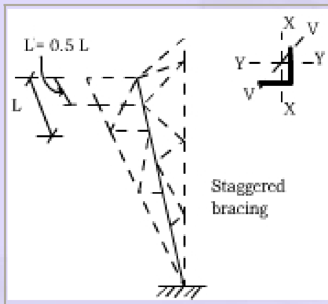
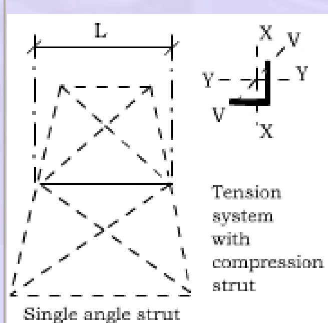
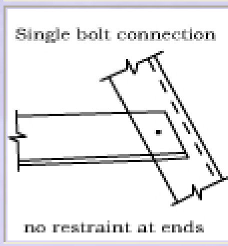
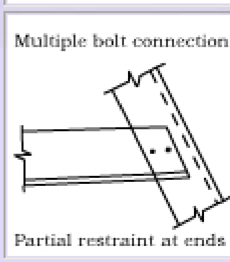
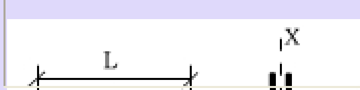
Figure 7.30 Nomogram showing the variation of the effective slenderness ratio kl / rL / r and the corresponding unit stress

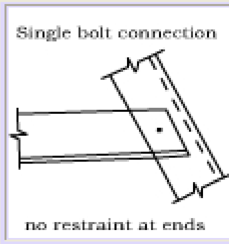
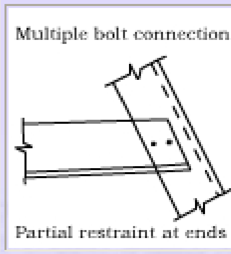
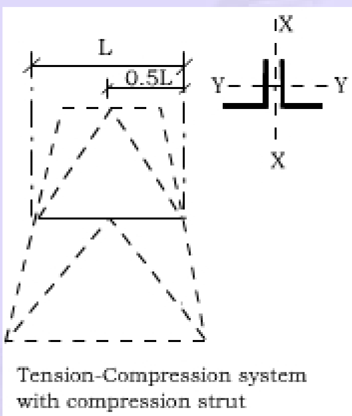
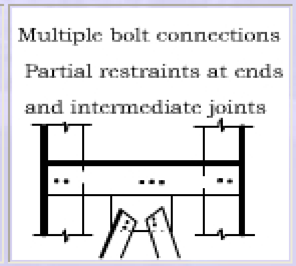
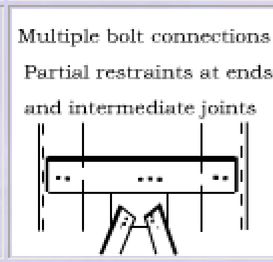
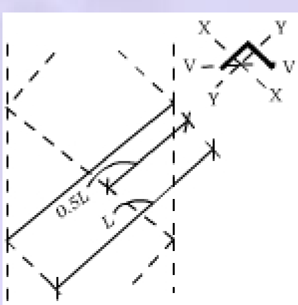
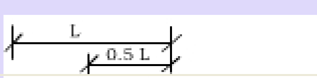
Figure 7.30 shows the variation of effective slenderness ratio KL / r with L / r of the member for the different cases of end restraint for leg and bracing members.

The value of KL / r to be chosen for estimating the unit stress on the compression strut depends on the following factors:

1. the type of bolted connection
2. the length of the member
3. the number of bolts used for the connection, i.e., whether it is a single-bolted or mul- tiple-bolted connection
4. the effective radius of gyration

Table 7.31 shows the identification of cases mentioned in Table 7.30 and Figure 7.30 for leg and bracing members normally adopted. Eight different cases of bracing systems are discussed in Table 7.31.

Sl. No 1	Member 2	Method of loading 3	Rigidity of joint 4	L/r ratio 5	Limiting values of L/r 6	Categorisation of member 7	KL/r 8
1		Concentric	No restraint at ends	L/r_w	0 to 120	Case (a)	L/r
					120 to 150	Case (e)	L/r
2		Concentric	No restraint at ends	L/r_{xx} or L/r_{yy} or $0.5L/r_w$	0 to 120	Case (a)	L/r
				L/r_{xx} or L/r_{yy} or $0.5L/r_w$	120 to 150	Case (e)	L/r
3		eccentric	unsupported panel-no restraint at ends	L/r_w	0 to 120	Case (d)	60 +0.5L/r
				L/r_w	120 to 200	Case (e)	L/r
				L/r_w	120 to 250	Case (g)	46.2 + 0.615L/r
4		concentric	No restraint at ends	max of L/r_{xx} or L/r_{yy}	0 to 120	Case (b)	L/r

				max of L/r_{xx} or L/r_{yy}	120 to 200	Case (e)	L/r
				max of L/r_{xx} or L/r_{yy}	120 to 250	Case (g)	46.2 + 0.615L/r
5		concentric at ends and eccentric at intermediate joints in both directions		$0.5L/r_{yy}$ or L/r_{xx}	0 to 120	Case (e)	30 + 0.75L/r
		concentric at ends and intermediate joints		$0.5L/r_{yy}$ or L/r_{xx}	0 to 120	Case (a)	L/r
		concentric at ends	Multiple bolt connections Partial restraints at ends and intermediate joints	$0.5L/r_{yy}$ or L/r_{xx}	120 to 250	Case (g)	46.2 + 0.615L/r
6		eccentric (single angle)	Single bolt No restraint at ends	$0.5L/r_w$ or $0.75L/r_{xx}$	0 to 120	Case (c)	30 + 0.75L/r
			Single bolt No restraint at ends	$0.5L/r_w$ or $0.75L/r_{xx}$	120 to 200	Case (e)	L/r
		concentric (Twin angle)	Multiple bolt connections Partial restraints at ends and intermediate joints	$0.5L/r_w$ or $0.75L/r_{xx}$	120 to 250	Case (g)	46.2 + 0.615L/r
7		eccentric (single angle)	Single or multiple bolt connection	$0.5L/r_w$ or L/r_{xx}	0 to 120	Case (g)	60 + 0.5L/r

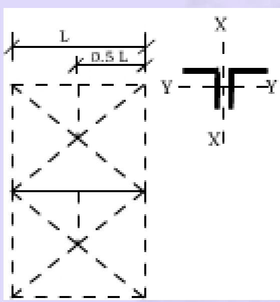
			Single bolt connection, no restraint at ends and at intermediate joints.	$0.5L/r_w$ or L/r_{xx}	120 to 200	Case (e)	L/r
			Multiple bolt at ends and single bolt at intermediate joints	$0.5L/r_w$	120 to 225	Case (f)	28.6 + .762L/r
			Multiple bolt at ends and at intermediate joints Partial restraints at both ends	L/r_{xx}	120 to 250	Case (g)	46.2 + 0.615L/r
			Partial restraints at ends and at intermediate joints	$0.5L/r_w$ or L/r_{xx}	120 to 250	Case (g)	46.2 + 0.615L/r
8		eccentric (single angle)	Single or multiple bolt connection	$0.5L/r_{yy}$ or L/r_{xx}	0 to 120	Case (a)	L/r
			Single bolt connection, no restraint at ends and at intermediate joints.	$0.5L/r_{yy}$ or L/r_{xx}	120 to 200	Case (e)	L/r
			Multiple bolt at ends and single bolt at intermediate joints	$0.5L/r_{yy}$	120 to 200	Case (f)	28.6 + .762L/r
			Multiple bolt connection Partial restraints at both ends	L/r_{xx}	120 to 250	Case (g)	46.2 + 0.615L/r
			Partial restraints at ends and at intermediate joints	$0.5L/r_{yy}$ or L/r_{xx}	120 to 250	Case (g)	46.2 + 0.615L/r

Table 7.31 Categorisation of members according to eccentricity of loading and end restraint conditions

Euler failure load

Euler determined the failure load for a perfect strut of uniform cross-section with hinged ends. The critical buckling load for this strut is given by:

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EA}{\left(\frac{L}{r}\right)^2} \qquad \textit{(7.30)}$$

The effective length for a strut with hinged ends is L.

At values less than $\pi^2 EL/L^2$ the strut is in a stable equilibrium. At values of P greater than $\pi^2 EL/L^2$ the strut is in a condition of unstable equilibrium and any small disturbance produces final collapse. This is, however, a hypothetical situation because all struts have some initial imperfections and thus the load on the strut can never exceed $\pi^2 EL/L^2$. If the thrust P is plotted against the lateral displacement Δ at any section, the P - Δ relationship for a perfect strut will be as shown in Figure 7.31 (a).

In this figure, the lateral deflections occurring after reaching critical buckling load are shown, that is $P_{cr} \geq \pi^2 EI / L^2$, When the strut has small imperfections, displacement is possible for all values of P and the condition of neutral equilibrium P = $\pi^2 EL/L^2$is never attained. All materials have a limit of proportionality. When this is reached, the flexural stiffness decreases initiating failure before P = $\pi^2 EL/L^2$ is reached (Figure 7.31 (b))

Empirical formulae

The following parameters influence the safe compressive stress on the column:

1. Yield stress of material
2. Initial imperfectness
3. (L/r) ratio
4. Factor of safety
5. End fixity condition
6. (b/t) ratio (Figure 7.31)) which controls flange buckling

Figure 7.31 (d) shows a practical application of a twin-angle strut used in a typical bracing system.

Taking these parameters into consideration, the following empirical formulae have been used by different authorities for estimating the safe compressive stress on struts:

1. Straight line formula
2. Parabolic formula
3. Rankine formula
4. Secant or Perry's formula

These formulae have been modified and used in the codes evolved in different countries.

IS: 802 (Part I) -1977 gives the following formulae which take into account all the parameters listed earlier.

For the case b / t $\leq$ 13 (Figure 7.30 (c)),

$$F_a = \left\{ 2600 - \frac{\left(\frac{KL}{r}\right)^2}{12} \right\} kg/cm^2 \qquad \textbf{\textit{(7.31)}}$$

Where KL / r $\leq$ 120

$$F_a = \frac{20 \, x \, 10^6}{\left(\frac{KL}{r}\right)^2} kg/cm^2 \qquad \textbf{\textit{(7.32)}}$$

Where KL / r > 120

$$F_{cr} = 4680 - 160(b / t)\ kg/cm^2$$

Where 13 < b/t < 20 ***(7.33)***

$$F_{cr} = \frac{590000}{\left(\frac{b}{t}\right)^2} kg / cm^2$$

Where b / t > 20 ***(7.34)***

Where

F_a = buckling unit stress in compression,

F_{cr} = limiting crippling stress because of large value of b / t,

b = distance from the edge of fillet to the extreme fibre, and

t = thickness of material.

Equations (7.31) and (7.32) indicate the failure load when the member buckles and Equations (7.33) and (7.34) indicate the failure load when the flange of the member fails.

Figure 7.30 gives the strut formula for the steel with a yield stress of 2600 kg/sq.cm. with respect to member failure. The upper portion of the figure shows the variation of unit stress with KL/r and the lower portion variation of KL/r with L/r. This figure can be used as a nomogram for estimating the allowable stress on a compression member.

An example illustrating the procedure for determining the effective length, the corresponding slenderness ratio, the permissible unit stress and the compressive force for a member in a tower is given below.

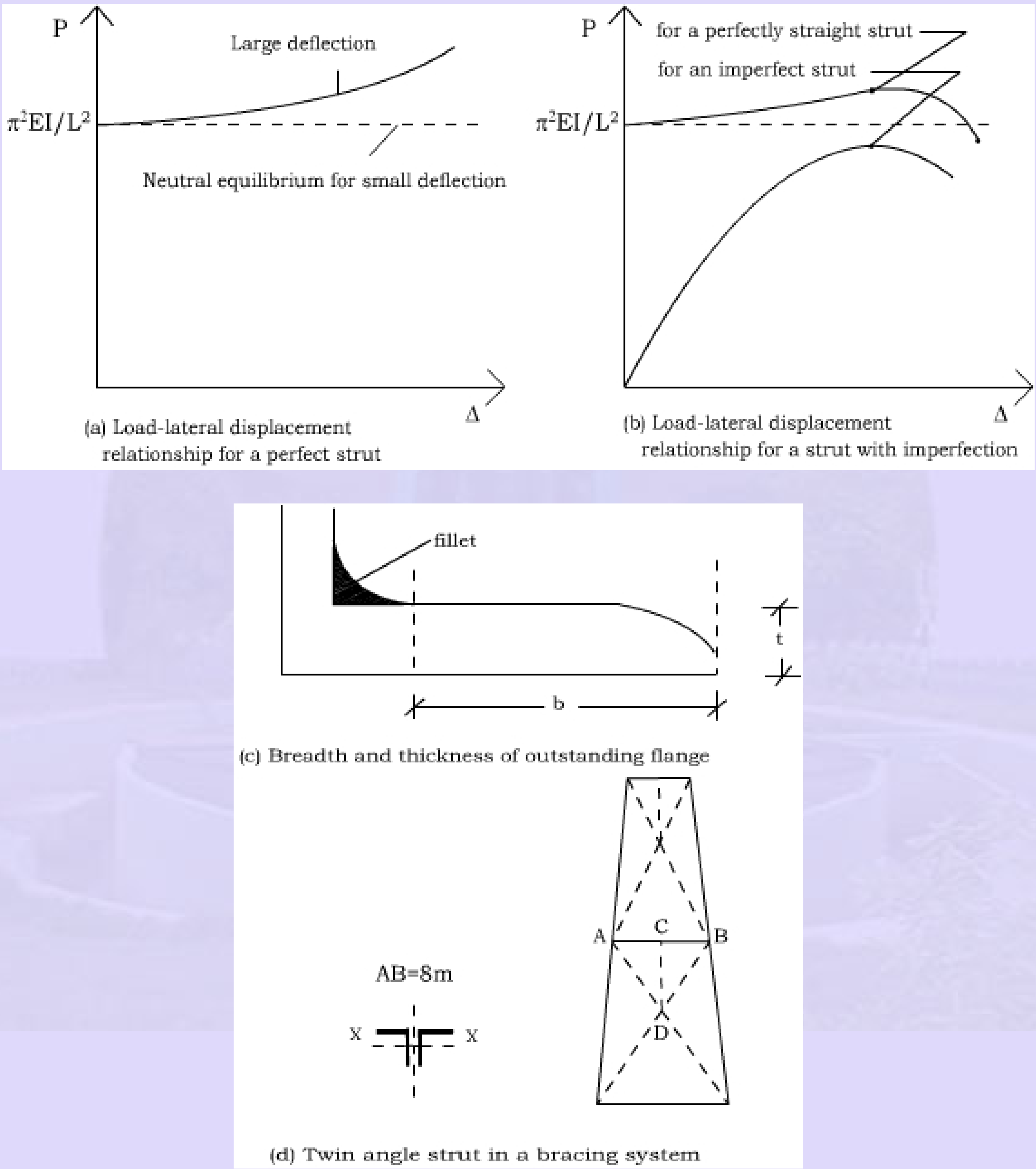


Figure 7.31

Example

Figure 7.31 (d) shows a twin angle bracing system used for the horizontal member of length L = 8 m. In order to reduce the effective length of member AB, single angle CD has been connected to the system. AB is made of two angles 100 x 100mm whose properties are given below:

r_{xx} = 4.38 cm

r_{yy} = 3.05 cm

Area = 38.06 sq.cm.

Double bolt connections are made at A, Band C. Hence it can be assumed that the joints are partially restrained. The system adopted is given at SL. No.8 in Table 7.31. For partial restraint at A, B and C,

$L/r = 0.5\, L/r_{yy}$ or L/r_{xx}

= 0.5 x 800/3.05 or 800/4.38

= 131.14 or 182.64

The governing value of L/r is therefore182.64, which is the larger of the two values obtained. This value corresponds to case (g) for which KL/r

= 46.2 + 0.615L/r

= 158.52

Note that the value of KL/r from the curve is also 158.52 (Figure 7.30). The corresponding stress from the curve above is 795 kg/cm^2, which is shown dotted in the nomogram. The value of unit stress can also be calculated from equation (7.32). Thus,

$$F_a = \frac{20 \times 10^6}{\left(\frac{KL}{r}\right)^2} kg/cm^2$$

$= 20 \times 10^6 / 158.52 \times 158.52$

$= 795\ kg/cm^2$.

The safe compression load on the strut AB is therefore

F = 38.06 x 795

= 30,257 kg

7.4.2 Computer-aided design

Two computer-aided design methods are in vogue, depending on the computer memory. The first method uses a fixed geometry (configuration) and minimizes the weight of the tower, while the second method assumes the geometry as unknown and derives the minimization of weight.

Method 1: Minimum weight design with assumed geometry

Power transmission towers are highly indeterminate and are subjected to a variety of loading conditions such as cyclones, earthquakes and temperature variations.

The advent of computers has resulted in more rational and realistic methods of structural design of transmission towers. Recent advances in optimisation in structural design have also been incorporated into the design of such towers.

While choosing the member sizes, the large number of structural connections in three dimensions should be kept in mind. The selection of members is influenced by their position in relation to the other members and the end connection conditions. The leg sections which carry different stresses at each panel may be assigned different sizes at various levels; but consideration of the large number of splices involved indicates that it is usually more economical and convenient, even though heavier, to use the same section for a number of panels. Similarly, for other members, it may be economical to choose a section of relatively large flange width so as to eliminate gusset plates and correspondingly reduce the number of bolts.

In the selection of structural members, the designer is guided by his past experience gained from the behavior of towers tested in the test station or actually in service. At certain critical locations, the structural members are provided with a higher margin of safety, one example being the horizontal members where the slope of the tower changes and the web members of panels are immediately below the neckline.

Optimisation

Many designs are possible to satisfy the functional requirements and a trial and error procedure may be employed to choose the optimal design. Selection of the best geometry of a tower or the member sizes is examples of optimal design procedures. The computer is best suited for finding the optimal solutions. Optimisation then becomes an automated design procedure, providing the optimal values for certain design quantities while considering the design criteria and constraints.

Computer-aided design involving user-ma- chine interaction and automated optimal design, characterized by pre-programmed logical decisions, based upon internally stored information, are not mutually exclusive, but complement each other. As the techniques of interactive computer-aided design develop, the need to employ standard routines for automated design of structural subsystems will become increasingly relevant.

The numerical methods of structure optimisation, with application of computers, automatically generate a near optimal design in an iterative manner. A finite number of variables has to be established, together with the constraints, relating to these variables. An initial guess-solution is used as the starting point for a systematic search for better designs and the process of search is terminated when certain criteria are satisfied.

Those quantities defining a structural system that are fixed during the automated design are- called pre-assigned parameters or simply parameters and those quantities that are not pre-assigned are called design variables. The design variables cover the material properties, the topology of the structure, its geometry and the member sizes. The assignment of the parameters as well as the definition of their values is made by the designer, based on his experience.

Any set of values for the design variables constitutes a design of the structure. Some designs may be feasible while others are not. The restrictions that must be satisfied in order to produce a feasible design are called constraints. There are two kinds of constraints: design constraints and behavior constraints. Examples of design constraints are minimum thickness of a member, maximum height of a structure, etc. Limitations on the maximum stresses, displacements or buck- ling strength are typical examples of behavior constraints. These constraints are expressed ma- thematically as a set of inequalities:

$$g_j(\{X\}) \le 0 \qquad j = 1, 2, ..., m \qquad \textbf{(7.35a)}$$

Where {X} is the design vector, and

m is the number of inequality constraints.

In addition, we have also to consider equality constraints of the form

$$h_j(\{X\}) \le 0 \qquad j = 1, 2, ..., k \qquad \textbf{(7.35b)}$$

Where k is the number of equality constraints.

Example

The three bar truss example first solved by Schmit is shown in Figure 7.32. The applied loadings and the displacement directions are also shown in this figure.

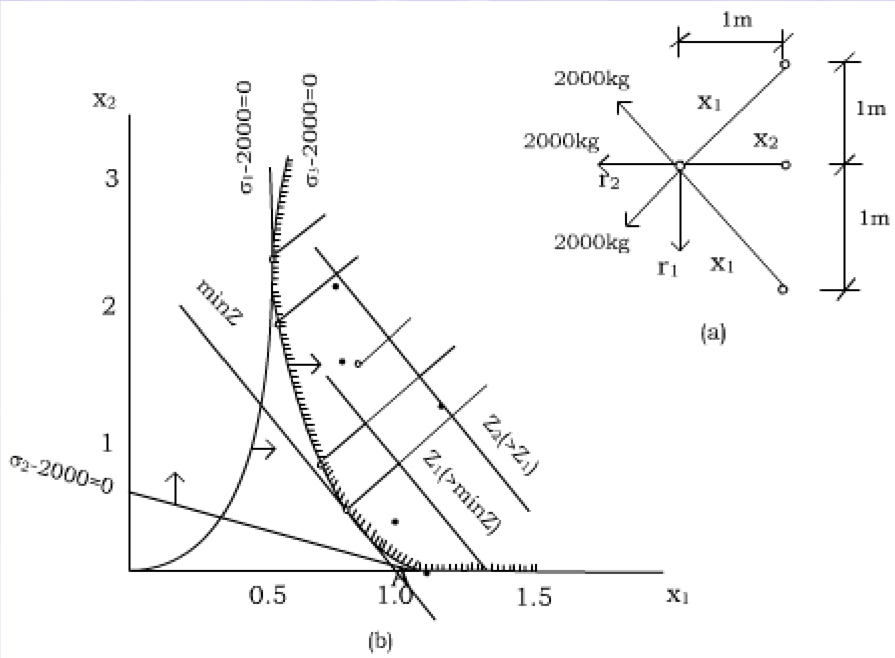


Figure 7.32 Two dimensional plot of the design variables X_1 and X_2

1. **Design constraints:** The condition that the area of members cannot be less than zero can be expressed as

$$g_1 \equiv -X_1 \le 0$$
$$g_2 \equiv -X_2 \le 0$$

2. Behaviour constraints: The three members of the truss should be safe, that is, the stresses in them should be less than the allowable stresses in tension (2,000 kg/cm^2) and compression (1,500kg/cm^2). This is expressed as

$$g_3 \equiv \sigma_1 - 2,000 \le 0$$ Tensile stress limitation in member 1

$$g_4 \equiv -\sigma_1 - 1,500 \le 0$$

$$g_5 \equiv \sigma_2 - 2,000 \le 0$$
$$g_6 \equiv -\sigma_2 - 1,500 \le 0$$
$$g_7 \equiv \sigma_3 - 2,000 \le 0$$
$$g_8 \equiv -\sigma_3 - 1,500 \le 0$$

Compressive stress limitation in member 2 and so on

3. Stress force relationships: Using the stress-strain relationship σ = [E] {Δ} and the force-displacement relationship F = [K] {Δ}, the stress-force relationship is obtained as {s} = [E] [K]$^{-1}$[F] which can be shown as

$$\sigma_1 = 2000\left(\frac{X_2 + \sqrt{2}X_1}{2X_1X_2 + \sqrt{2}X_1{}^2}\right)$$
$$\sigma_2 = 2000\left(\frac{\sqrt{2}X_1}{2X_1X_2 + \sqrt{2}X_1{}^2}\right)$$
$$\sigma_1 = 2000\left(\frac{X_2}{2X_1X_2 + \sqrt{2}X_1{}^2}\right)$$

4. Constraint design inequalities: Only constraints g_3, g_5, g_8 will affect the design. Since these constraints can now be expressed in terms of design variables X_1 and X_2 using the stress force relationships derived above, they can be represented as the area on one side of the straight line shown in the two-dimensional plot (Figure 7.32 (b)).

Design space

Each design variable X_1, X_2 ...is viewed as one- dimension in a design space and a particular set of variables as a point in this space. In the general case of n variables, we have an n-dimensioned space. In the example where we have only two variables, the space reduces to a plane figure shown in Figure 7.32 (b). The arrows indicate the inequality representation and the shaded zone shows the feasible region. A design falling in the feasible region is an unconstrained design and the one falling on the boundary is a constrained design.

Objective function

An infinite number of feasible designs are possible. In order to find the best one, it is necessary to form a function of the variables to use for comparison of feasible design alternatives. The objective (merit) function is a function whose least value is sought in an optimisation procedure. In other words, the optimization problem consists in the determination of the vector of variables X that will minimise a certain given objective function:

$$Z = F(\{X\}) \qquad \textbf{\textit{7.35(c)}}$$

In the example chosen, assuming the volume of material as the objective function, we get

$$Z = 2(141\ X_1) + 100\ X_2$$

The locus of all points satisfying F ({X}) = constant, forms a straight line in a two-dimensional space. In this general case of n-dimensional space, it will form a surface. For each value of constraint, a different straight line is obtained. Figure 7.32 (b) shows the objective function contours. Every design on a particular contour has the same volume or weight. It can be seen that the minimum value of F ({X}) in the feasible region occurs at point A.

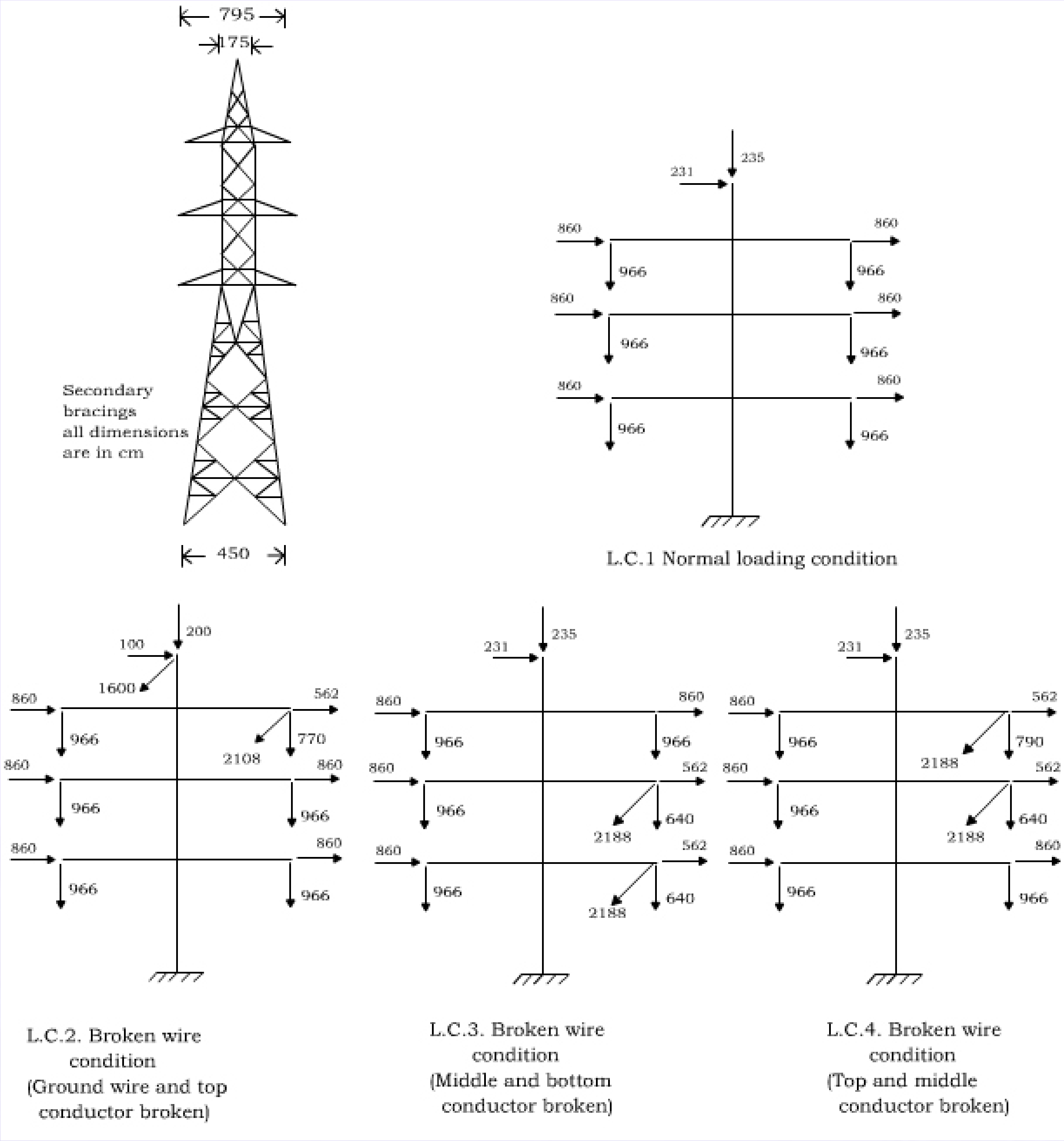


Figure 7.33 Configuration and loading condition for the example tower

There are different approaches to this problem, which constitute the various methods of optimization. The traditional approach searches the solution by pre-selecting a set of critical constraints and reducing the problem to a set of equations in fewer variables. Successive reanalysis of the structure for improved sets of constraints will tend towards the solution. Different re-analysis methods can be used, the iterative methods being the most attractive in the case of towers.

Optimality criteria

An interesting approach in optimization is a process known as optimality criteria. The approach to the optimum is based on the assumption that some characteristics will be attained at such optimum. The well-known example is the fully stressed design where it is assumed that, in an optimal structure, each member is subjected to its limiting stress under at least one loading condition.

The optimality criteria procedures are useful for transmission lines and towers because they constitute an adequate compromise to obtain practical and efficient solutions. In many studies, it has been found that the shape of the objective function around the optimum is flat, which means that an experienced designer can reach solutions, which are close to the theoretical optimum.

Mathematical programming

It is difficult to anticipate which of the constraints will be critical at the optimum. Therefore, the use of inequality constraints is essential for a proper formulation of the optimal design problem.

The mathematical programming (MP) methods are intended to solve the general optimisation problem by numerical search algorithms while being general regarding the objective function and constraints. On the other hand, approximations are often required to be efficient on large practical problems such as tower optimisation.

Optimal design processes involve the minimization of weight subject to certain constraints. Mathematical programming methods and structural theorems are available to achieve such a design goal.

Of the various mathematical programming methods available for optimisation, the linear programming method is widely adopted in structural engineering practice because of its simplicity. The objective function, which is the minimisation of weight, is linear and a set of constraints, which can be expressed by linear equations involving the unknowns (area, moment of inertia, etc. of the members), are used for solving the problems. This can be mathematically expressed as follows.

Suppose it is required to find a specified number of design variables x_1, x_2.....x_n such that the objective function

$$Z = C_1\, x_1 + C_2\, x_2 + \ldots.C_n\, x_n$$

is minimised, satisfying the constraints

$$\begin{aligned} & a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots.a_{1n}x_n \le b_1 \\ & a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots.a_{2n}x_n \le b_2 \\ & . \\ & . \\ & . \\ & a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots.a_{mn}x_n \le b_m \end{aligned} \qquad \textbf{\textit{(7.36)}}$$

The simplex algorithm is a versatile procedure for solving linear programming (LP) problems with a large number of variables and constraints.

The simplex algorithm is now available in the form of a standard computer software package, which uses the matrix representation of the variables and constraints, especially when their number is very large.

The equation (7.36) is expressed in the matrix form as follows:

Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ - \\ - \\ x_n \end{Bmatrix}$ which minimises the objective function

$$f(x) = \sum_{i-1}^{n} C_i x_i \qquad \textbf{\textit{(7.37)}}$$

subject to the constraints,

$$\sum_{k-1}^{n} a_{jk} x_k = b_j, \quad j = 1, 2, \ldots m \qquad \textbf{\textit{(7.38)}}$$
$$\text{and} x_i \geq 0, \quad i = 1, 2, \ldots n$$

where C_i, a_{jk} and b_j are constants.

The stiffness method of analysis is adopted and the optimisation is achieved by mathematical programming.

The structure is divided into a number of groups and the analysis is carried out group wise. Then the member forces are determined. The critical members are found out from each group. From the initial design, the objective function and the constraints are framed. Then, by adopting the fully stressed design (optimality criteria) method, the linear programming problem is solved and the optimal solution found out. In each group, every member is designed for the fully stressed condition and the maximum size required is assigned for all the members in that group. After completion of the design, one more analysis and design routine for the structure as a whole is completed for alternative cross-sections.

Example

A 220 k V double circuit tangent tower is chosen for study. The basic structure, section plan at various levels and the loading conditions are tentatively fixed. The number of panels in the basic determinate structure is 15 and the number of members is 238. Twenty standard sections have been chosen in the increasing order of weight. The members have been divided into eighteen groups, such as leg groups, diagonal groups and horizontal groups, based on various panels of the tower. For each group a section is specified.

Normal loading conditions and three broken- wire conditions has been considered. From the vertical and horizontal lengths of each panel, the lengths of the members are calculated and the geometry is fixed. For the given loading conditions, the forces in the various members are computed, from which the actual stresses are found. These are compared with allowable stresses and the most stressed member (critical) is found out for each group. Thereafter, an initial design is evolved as a fully stressed design in which critical members are stressed up to an allowable limit. This is given as the initial solution to simplex method, from which the objective function, namely, the weight of the tower, is formed. The initial solution so obtained is sequentially improved, subject to the constraints, till the optimal solution is obtained.

In the given solution, steel structural angles of weights ranging from 5.8 kg/m to 27.20 kg/m are utilised. On the basis of the fully stressed design, structural sections of 3.4 kg/m to 23.4 kg/m are indicated and the corresponding weight is 5,398 kg. After the optimal solution, the weight of the tower is 4,956 kg, resulting in a saving of about 8.1 percent.

Method 2: Minimum weight design with geometry as variable

In Method 1, only the member sizes were treated as variables whereas the geometry was assumed as fixed. Method 2 treats the geometry also as a variable and gets the most preferred geometry. The geometry developed by the computer results in the minimum weight of tower for any practically acceptable configuration. For solution, since an iterative procedure is adopted for the optimum structural design, it is obvious that the use of a computer is essential.

The algorithm used for optimum structural design is similar to that given by Samuel L. Lipson which presumes that an initial feasible configuration is available for the structure. The structure is divided into a number of groups and the externally applied loadings are obtained. For the given configuration, the upper limits and the lower limits on the design variables, namely, the joint coordinates are fixed. Then (k-1) new configurations are generated randomly as

$$x_{ij} = l_i + r_{ij}(u_i - l_i) \quad \textbf{(7.39)}$$

$$i = 1, 2 \ldots n$$

$$j = 1, 2 \ldots k$$

where k is the total number of configurations in the complex, usually larger than (n + 1), where n is the number of design variables and r_{ij} is the random number for the i^{th} coordinate of the j^{th} point, the random numbers having a uniform distribution over the interval 0 to 1 and u_i is the upper limit and L_i is the lower limit of the i^{th} independent variable.

Thus, the complex containing k number of feasible solutions is generated and all these configurations will satisfy the explicit constraints, namely, the upper and lower bounds on the design variables. Next, for all these k configurations, analysis and fully stressed designs are carried out and their corresponding total weights determined. Since the fully stressed design concept is an eco nomical and practical design, it is used for steel area optimisation. Every area optimisation problem is associated with more than one analysis and design. For the analysis of the truss, the matrix method described in the previous chapter has been used. Therefore, all the generated configurations also satisfy the implicit constraints, namely, the allowable stress constraints.

From the value of the objective function (total weight of the structure) of k configurations, the vector, which yields the maximum weight, is searched and discarded, and the centroid c of each joint of the k-1 configurations is determined from

$$x_{ic} = \frac{1}{K-1}\left\{K\sum_{j-1}\left(x_{ij}\right) - x_{iw}\right\} \qquad \textbf{\textit{(7.40)}}$$

i = 1, 2, 3 ... n

in which x_{ic} and x_{iw} are the i^{th} coordinates of the centroid c and the discarded point w.

Then a new point is generated by reflecting the worst point through the centroid, x_{ic}

That is, $x_{iw} = x_{ic} + \alpha (x_{ic} - x_{iw})$

(7.41)

i = 1,2,..... n where α is a constant.

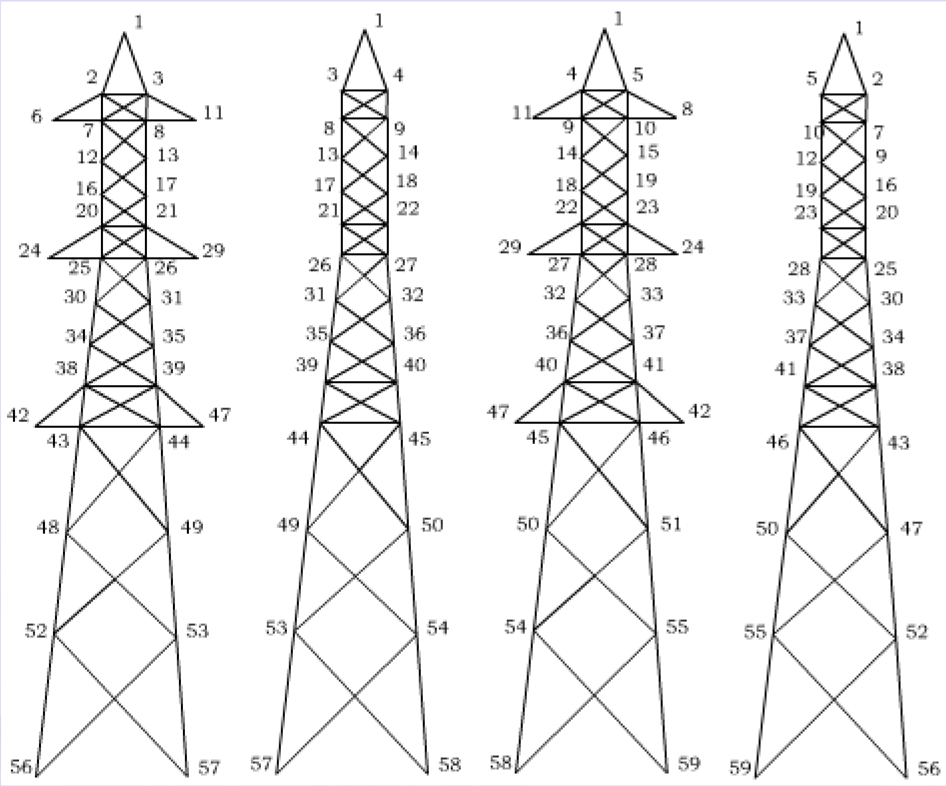


Figure 7.34 Node numbers

This new point is first examined to satisfy the explicit constraints. If it exceeds the upper or lower bound value, then the value is taken as the corresponding limiting value, namely, the upper or lower bound. Now the area optimisation is carried out for the newly generated configuration and the functional value (weight) is determined. If this functional value is better than the second worst, the point is accepted as an improvement and the process of developing the new configuration is repeated as mentioned earlier. Otherwise, the newly generated point is moved halfway towards the centroid of the remaining points and the area optimisation is repeated for the new configuration.

This process is repeated over a fixed number of iterations and at the end of every iteration, the weight and the corresponding configuration are printed out, which will show the minimum weight achievable within the limits (l and u) of the configuration.

Example

The example chosen for the optimum structural design is a 220 k V double-circuit angle tower. The tower supports one ground wire and two circuits containing three conductors each, in vertical configuration, and the total height of the tower is 33.6 metres. The various load conditions are shown in Figure 7.33.

The bracing patterns adopted are Pratt system and Diamond system in the portions above and below the bottom-most conductor respectively. The initial feasible configuration is shown on the top left corner of Figure 7.33. Except x, y and z coordinates of the conductor and the z coordinates of the foundation points, all the other joint coordinates are treated as design variables. The tower configuration considered in this example is restricted to a square type in the plan view, thus reducing the number of design variables to 25.

In the initial complex, 27 configurations are generated, including the initial feasible configuration. Random numbers required for the generation of these configurations are fed into the comJ7llter as input. One set containing 26 random numbers with uniform distribution over the interval 0 to 1 are supplied for each design variable. Figure 7.34 and Figure 7.35 show the node numbers and member numbers respectively.

The example contains 25 design variables, namely, the x and y coordinates of the nodes, except the conductor support points and the z coordinates of the support nodes (foundations) of the tower. 25 different sets of random numbers, each set containing 26 numbers, are read for 25 design variables. An initial set of27 configuration is generated and the number of iterations for the development process is restricted to 30. The weight of the tower for the various configurations developed during optimisation procedure is pictorially represented in Figure 7.36. The final configuration is shown in Figure 7.37a and the corresponding tower weight, including secondary bracings, is 5,648 kg.

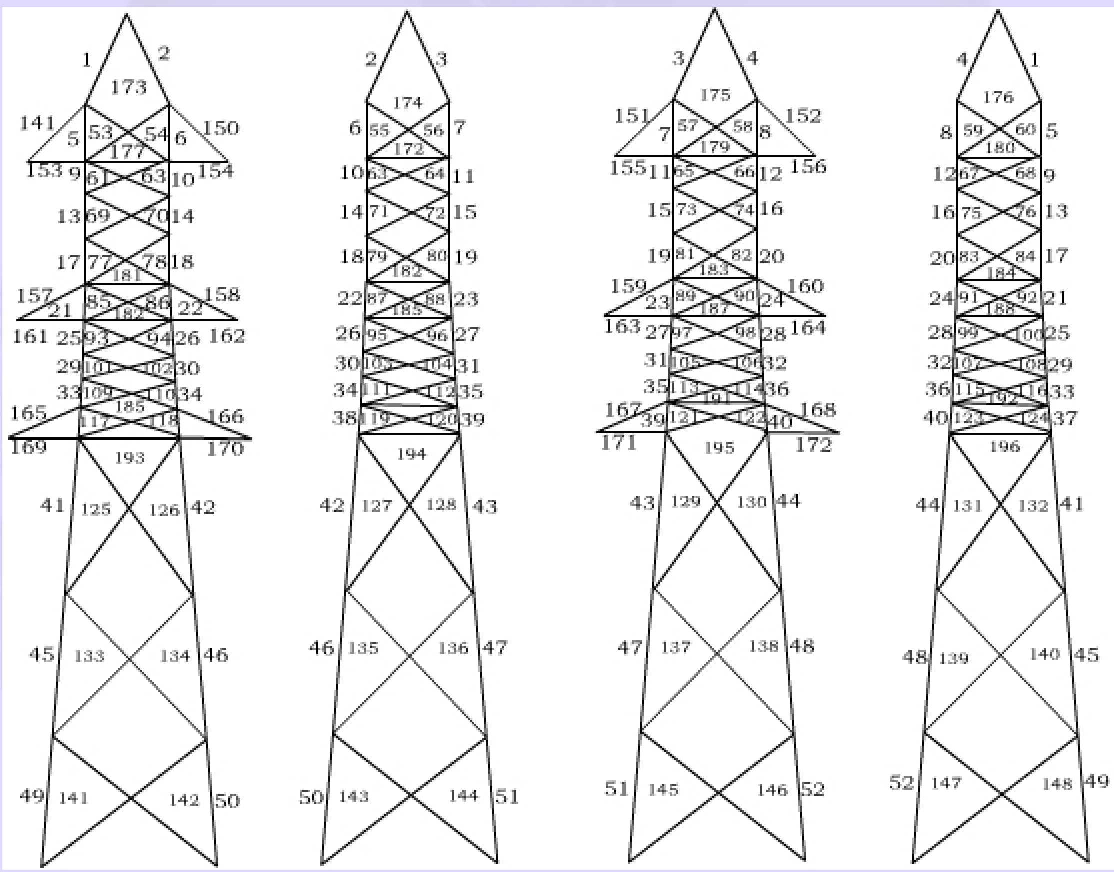


Figure 7.35 Member numbers

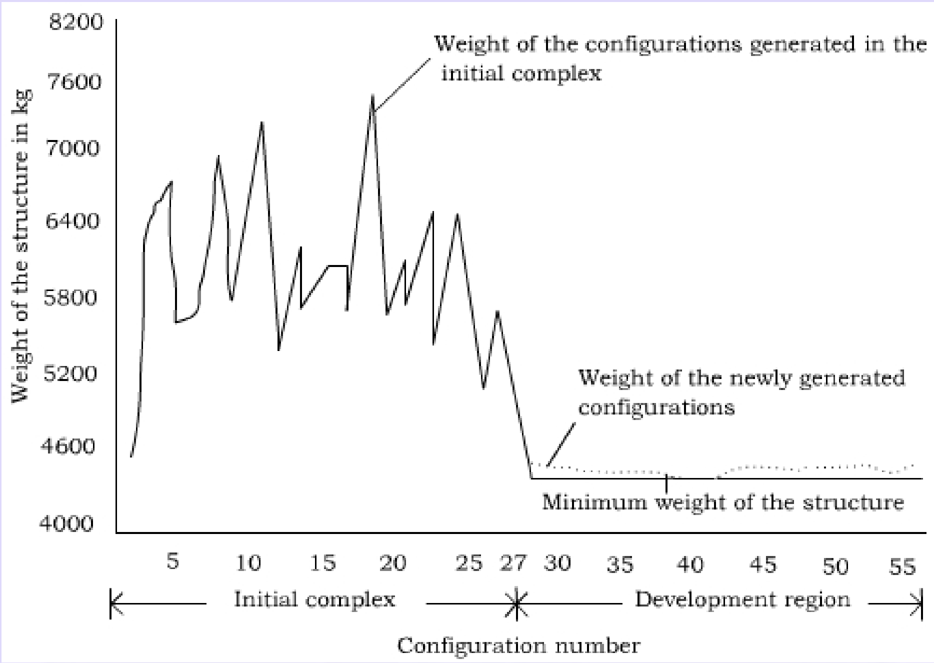


Figure 7.36 Tower weights for various configurations generated

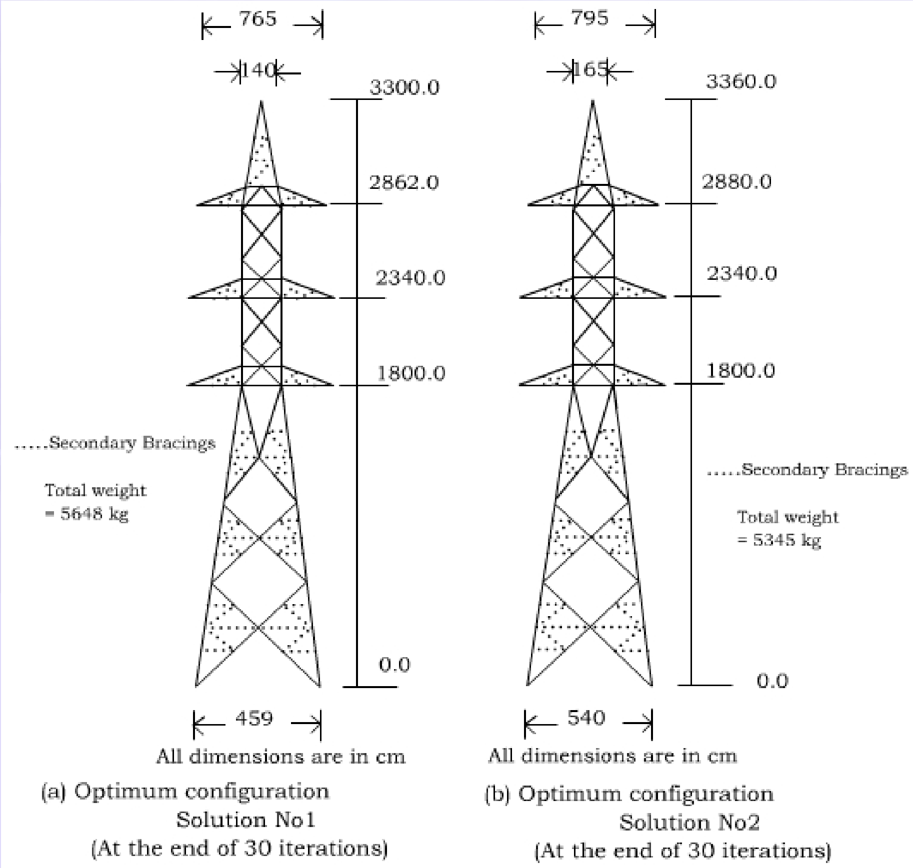


Figure 7.37

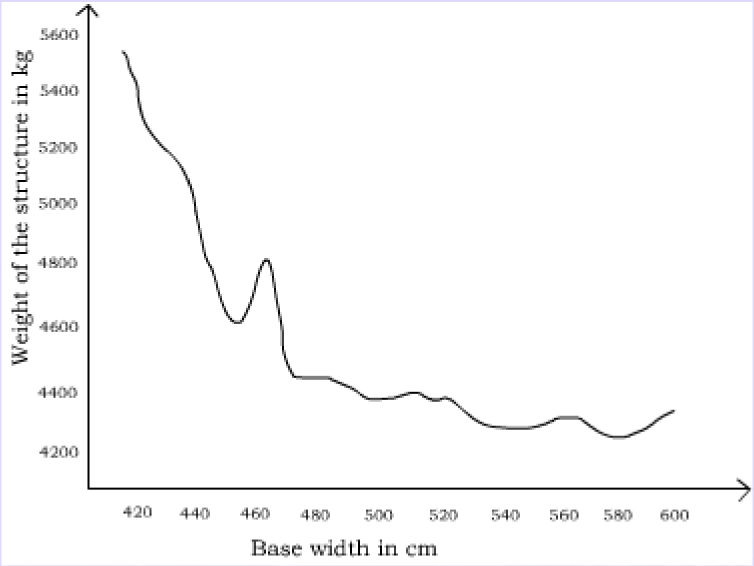


Figure 7.38 Variation of tower weight with base width

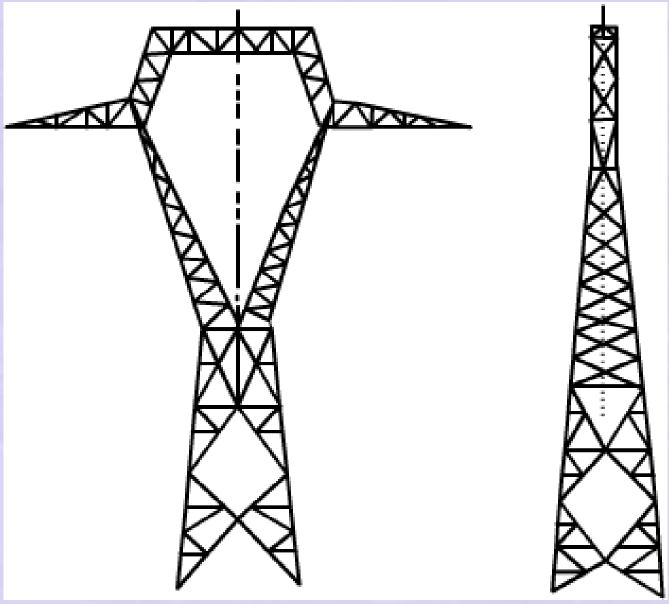


Figure 7.39 Tower geometry describing key joints and joints obtained from key joints

This weight can further be reduced by adopting the configuration now obtained as the initial configuration and repeating the search by varying the controlling coordinates x and z. For instance, in the present example, by varying the x coordinate, the tower weight has been reduced to 5,345 kg and the corresponding configuration is shown in Figure 7.37b. Figure 7.38 shows the variation of tower weight with base width.

In conclusion, the probabilistic evaluation of loads and load combinations on transmission lines, and the consideration of the line as a whole with towers, foundations, conductors and hardware, forming interdependent elements of the total sys- tem with different levels of safety to ensure a preferred sequence of failure, are all directed towards achieving rational behaviour under various uncertainties at minimum transmission line cost. Such a study may be treated as a global optimisation of the line cost, which could also include an examination of alternative uses of various types of towers in a family, materials to be employed and the limits to which different towers are utilised as discrete variables and the objective function as the overall cost.

7.4.3 Computer software packages

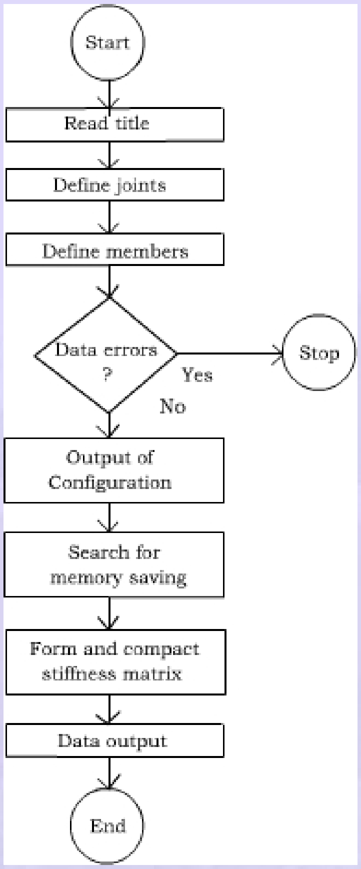


Figure 7.40 Flowchart for the development of tower geometry in the OPSTAR program

The general practice is to fix the geometry of the tower and then arrive at the loads for design purposes based on which the member sizes are determined. This practice, however, suffers from the following disadvantages:

1. The tower weight finally arrived at may be different from the assumed design weight.

2. The wind load on tower calculated using assumed sections may not strictly correspond to the actual loads arrived at on the final sections adopted.

3. The geometry assumed may not result in the economical weight of tower.

4. The calculation of wind load on the tower members is a tedious process.

Most of the computer software packages available today do not enable the designer to overcome the above drawbacks since they are meant essentially to analyse member forces.

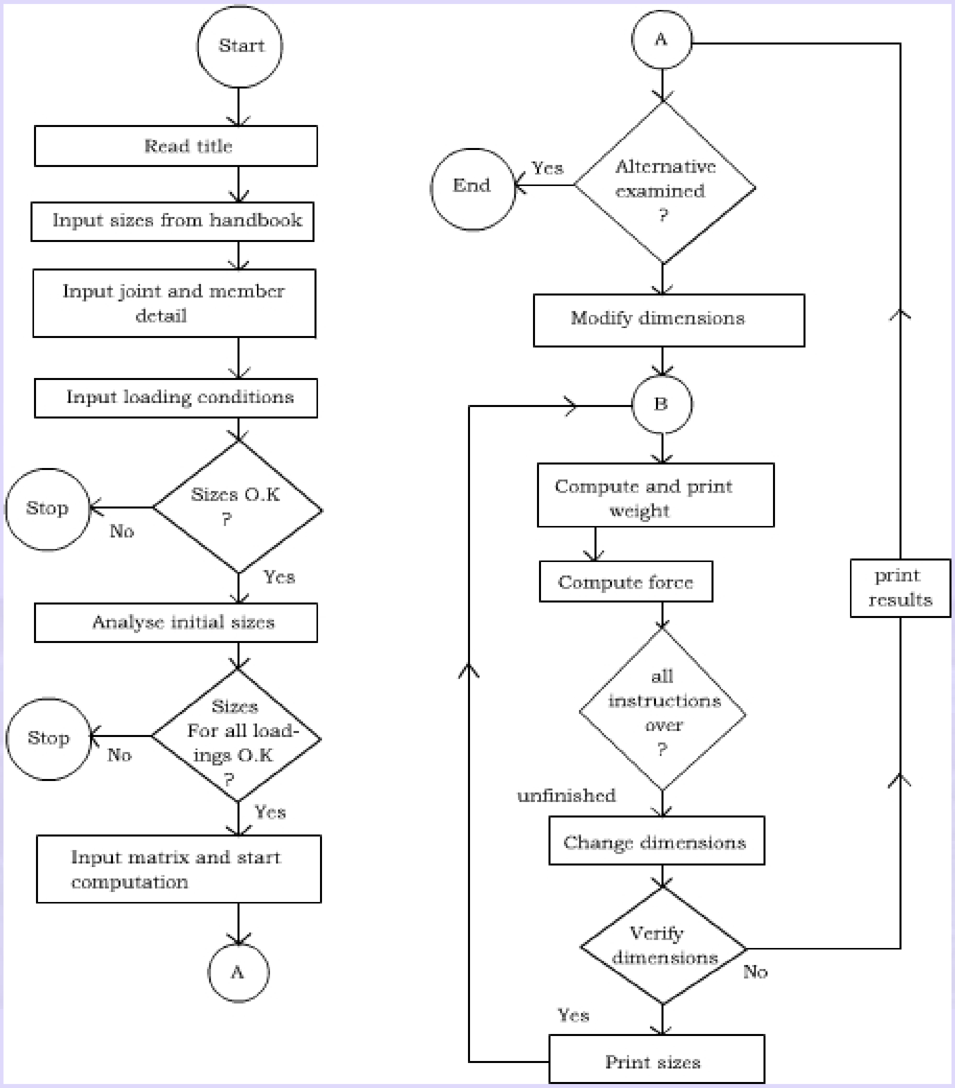


Figure 7.41 Flowchart for the solution sequence (opstar programme)

In Electricite de France (EDF), the OPSTAR program has been used for developing economical and reliable tower designs. The OPSTAR program optimises the tower member sizes for a fixed configuration and also facilitates the development of new configurations (tower outlines), which will lead to the minimum weight of towers. The salient features of the program are given below:

Geometry: The geometry of the tower is described by the coordinates of the nodes. Only the coordinates of the key nodes (8 for a tower in Figure 7.39) constitute the input. The computer generates the other coordinates, making use of symmetry as well as interpolation of the coordinates of the nodes between the key nodes. This simplifies and minimises data input and aids in avoiding data input errors.

Solution technique: A stiffness matrix approach is used and iterative analysis is performed for optimisation.

Description of the program: The first part of the program develops the geometry (coordinates) based on data input. It also checks the stability of the nodes and corrects the unstable nodes. The flow chart for this part is given in Figure 7.40.

The second part of the program deals with the major part of the solution process. The input data are: the list of member sections from tables in handbooks and is based on availability; the loading conditions; and the boundary conditions.

The solution sequence is shown in Figure 7.41. The program is capable of being used for either checking a tower for safety or for developing a new tower design. The output from the program includes tower configuration; member sizes; weight of tower; foundation reactions under all loading conditions; displacement of joints under all loading conditions; and forces in all members for all loading conditions.

7.4.4 Tower accessories

Designs of important tower accessories like Hanger, Step bolt, Strain plate; U-bolt and D-shackle are covered in this section. The cost of these tower accessories is only a very small fraction of the S overall tower cost, but their failure will render the tower functionally ineffective. Moreover, the towers have many redundant members whereas the accessories are completely determinate. These accessories will not allow any load redistribution, thus making failure imminent when they are overloaded. Therefore, it is preferable to have larger factors of safety associated with the tower accessories than those applicable to towers.

Hanger (Figure 7.42)

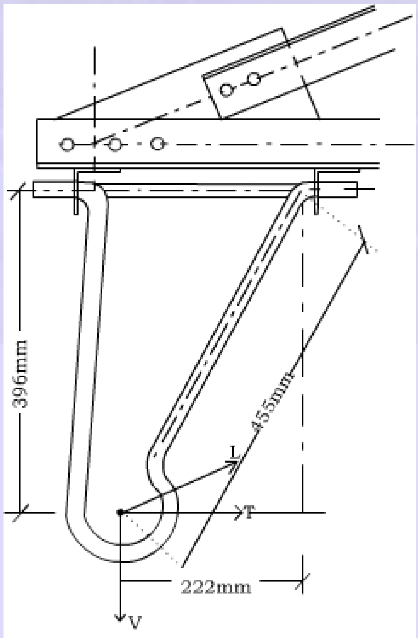


Figure 7.42 Hanger

The loadings coming on a hanger of a typical 132 kV double-circuit tower are given below:

Type of loading	NC	BWC
Transverse	480kg	250kg
Vertical	590kg	500kg
Longitudinal	-	2,475kg

Maximum loadings on the hanger will be in the broken-wire condition and the worst loaded member is the vertical member.

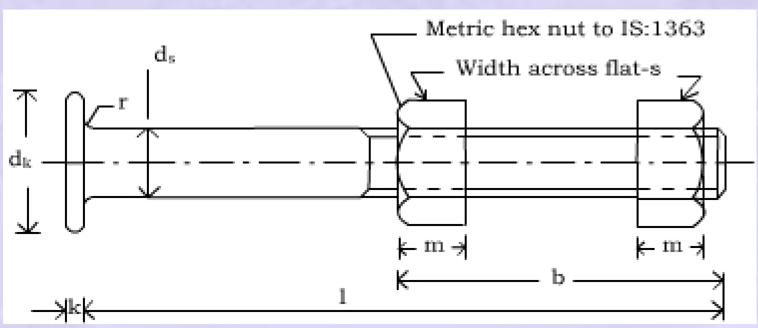
Diameter of the hanger leg = 21mm

Area = p x $(21)^2$ / 4 x 100 = 3.465 sq.cm.

Maximum allowable tensile stress for the steel used = 3,600 kg/cm^2

Allowable load = 3,600 x 3.465

= 12,474 kg.



<table>
<tr><td colspan="16">Dimensions</td></tr>
<tr><td>Nom bolt dia</td><td>threads</td><td colspan="2">Shank dia d_s</td><td colspan="2">Head dia d_k</td><td colspan="2">Head thickness k</td><td>Neck radius (app) r</td><td colspan="2">Bolt length l</td><td colspan="2">Thread length b</td><td colspan="2">Width across flats s</td><td>Nut thickness m</td></tr>
<tr><td colspan="16">Metric Serious (dimensions in mm before galvanising)</td></tr>
<tr><td rowspan="2">16</td><td rowspan="2">m 16</td><td rowspan="2">16</td><td>+1.10</td><td rowspan="2">35</td><td>+2</td><td rowspan="2">6</td><td>+1</td><td rowspan="2">3</td><td rowspan="2">175</td><td>+3</td><td rowspan="2">60</td><td>+5</td><td rowspan="2">24</td><td>+0</td><td rowspan="2">13 ± 0.55</td></tr>
<tr><td>-0.43</td><td>-0</td><td>-0</td><td>-0</td><td>-0</td><td>-0.84</td></tr>
</table>

Bolts	Nuts
1. Tensile strength - 400 N/mm^2 min.	1. Proof load stress - 400 N/mm^2
2. Brinell Hardness- HB 114/209	2. Brinell Hardness- HB 302 max
3. Cantilever load test - with 150kg	

Figure 7.43 Dimensions and mechanical properties of step bolts and nuts

Loads in the vertical leg

1. Transverse load (BWC) = 250 / 222 x 396

   = 446kg.

2. Longitudinal load = 2,475 kg.
3. Vertical load = 500 kg.

   Total = 3,421kg.

It is unlikely that all the three loads will add up to produce the tension in the vertical leg. 100 percent effect of the vertical load and components of longitudinal and transverse load will be acting on the critical leg to produce maximum force. In accordance with the concept of making the design conservative, the design load has been assumed to be the sum of the three and hence the total design load = 3,421 kg.

Factor of safety = 12,474 / 3,421 = 3.65 which is greater than 2, and hence safe

Step bolt (Figure 7.43)

Special mild steel hot dip galvanised bolts called step bolts with two hexagonal nuts each, are used to gain access to the top of the tower structure. The design considerations of such a step bolt are given below.

The total uniformly distributed load over the fixed length = 100 kg (assumed).

The maximum bending moment

100 x 13 / 2 = 650 kg cm.

The moment of inertia = p x 16^4 / 64 = 0.3218 cm^4

Maximum bending stress = 650 x 0.8 / 0.3218

= 1,616 kg/cm^2

Assuming critical strength of the high tensile steel = 3,600 kg/cm^2,

factor of safety = 3,600 / 1,616 = 2.23, which is greater than 2, and hence safe.

Step bolts are subjected to cantilever load test to withstand the weight of man (150kg).

Strain plate (Figure 7.44)

The typical loadings on a strain plate for a 132 kV double-circuit tower are given below:

Vertical load = 725kg

Transverse load = 1,375kg

Longitudinal load = 3,300kg

Bending moment due to vertical load = 725 x 8 / 2 = 2,900kg.cm.

I_{xx} = 17 x $(0.95)^3$ / 12 = 1.2146 cm^4

y (half the depth) = 0.475cm.

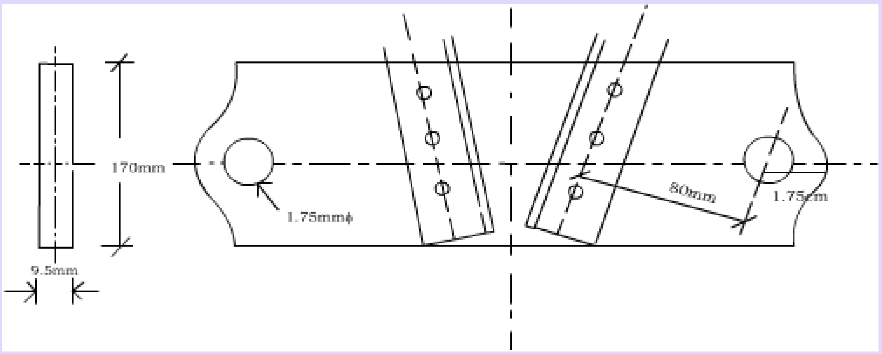


Figure 7.44 Strain plate

Section modulus Z_{xx} = 1.2146 / 0.475 = 2.5568

Bending stress f_{xx} = 2,900 / 2.5568 = 1,134 kg/cm^2

Bending moment due to transverse load = 1.375 x 8 / 2 = 5,500 kg.cm.

Actually the component of the transverse load in a direction parallel to the line of fixation should be taken into account, but it is safer to consider the full transverse load.

I_{yy} = 0.95 x 17^3 / 12 = 389 cm^4

Z_{yy} = 389 / 8.5 = 45.76

Bending stress f_{yy} = 5500 / 45.76 = 120 kg/cm^2

Total maximum bending stress

$f_{xx} + f_{yy}$ = 1,134 + 120 = 1,254 kg/cm^2

Direct stress due to longitudinal load = longitudinal load / Cross-sectional area

= 3,300 / 13.5 x 0.95

= 257.3 kg/cm^2

Check for combined stress

The general case for a tie, subjected to bending and tension, is checked using the following interaction relationship:

$$\frac{f_b}{F_b} + \frac{f_t}{F_T} \leq 1 \qquad \textbf{(7.36)}$$

Where f_t = actual axial tensile stress,

f_b = actual bending tensile stress

F_t = permissible axial tensile stress, and

F_b = permissible bending tensile stress.

Assuming F_t = 1,400 kg/cm^2 and F_b = 1,550 kg/cm^2. The expression reduces to

= 1,254 / 1,550 + 257.3 / 1,400

= 0.9927 <1, hence safe.

Check for the plate in shear

Length of the plate edge under shear = 1.75 cm

Area under shear = 2 x 1.75 x 0.95

= 3.325 sq.cm.

Shearing stress =3,300 / 3,325 = 992 kg/cm^2

Permissible shear stress = 1,000 kg/cm^2

Hence, it is safe in shear.

Check for the plate in bearing

Pin diameter = 19mm

Bearing area = 1.9 x 0.95 = 1.805 cm^2

Maximum tension in the conductor = 3,300 kg.

Bearing stress = = 1,828 kg/cm^2

Permissible bearing stress = 1860 kg/cm^2

Hence, it is safe in bearing.

Check for bolts in shear

Diameter of the bolt = 16mm

Area of the bolt = 2.01 sq.cm.

Shear stress = 3,300 / 3 x 2.01 = 549 kg/cm^2

Permissible shearing stress = 1,000 kg/cm^2

Hence, three 16mm diameter bolts are adequate.

U-bolt (Figure 7.45)

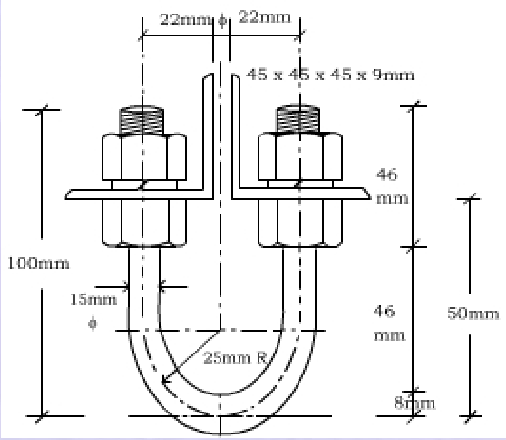


Figure 7.45 U-bolt

The loadings in a U-bolt for a typical 66 kV double circuit tower are given below:

	NC	BWC
Transverse load	= 216	108
Vertical load	= 273	227
Longitudinal load	= -	982

Permissible bending stress for mild steel = 1,500 kg/cm^2

Permissible tensile stress = 1,400 kg/cm^2

Let the diameter of the leg be 16mm.

The area of the leg = 2.01 sq.cm.

1. Direct stress due to vertical load = 273 / 2.01 x 2

$= 67.91 \text{ kg/cm}^2$

2. Bending due to transverse load (NC)

Bending moment = 216 x 5 = 1,080 kg.cm

Section Modulus = $2 \times \pi d^3 / 32 = 2 \times 3.14 \times 1.6^3 / 32$

= 0.804

Bending stress = 1,080 / 0.804

$= 1{,}343 \text{ kg/cm}^2 < 1{,}500 \text{ kg/cm}^2$

Hence safe.

3. Bending due to longitudinal load (BWC)

Bending moment = 982 x 5 = 4,910 kg.cm

$$I_{xx} = \left(\frac{\pi d^4}{64} + \frac{\pi d^2}{4} \text{ x } 2.5^2\right)^2 = 25.77 \text{cm}^4$$

y = 2.5 + 0.8

Bending stress = 4, 90 / 25.77 x (2.5 + 0.8) = 629 kg/cm^2

In the broken-wire condition total bending stress = 1,343 / 2 + 629

$= 1{,}300 \text{ kg/cm}^2$

Hence, the worst loading will occur during normal condition.

For safe design,

$$\frac{f_b}{F_b} + \frac{f_t}{F_t} \le 1$$

67.91 / 1400 + 1343 / 1500 = 0.9365 < 1

Hence safe.

Bearing strength of the angle-bolt connection

Safe bearing stress for the steel used = 4,725 kg/cm^2

Diameter of hole = 16mm + 1.5mm = 17.5mm

Thickness of the angle leg = 5mm

Under normal condition

Bearing stress = (216 + 273)/1.75 x 0.5 = 558.85 kg/cm^2

Factor of safety = 4,725 / 558.85 = 8.45

Under broken-wire condition

Bearing stress = (108+227+982) / 1.75 x 0.5

= 1,505.14 kg/cm^2

Therefore, factor of safety = 4,725 /1,505.14 = 3.13

Hence safe.

D-shackle (Figure 7.46)

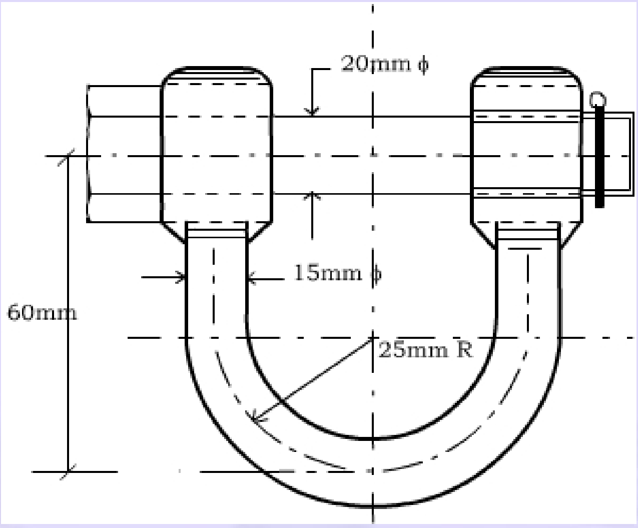


Figure 7.46 D-Shackle

The loadings for a D-shackle for a 132 kV single circuit tower are given below:

	NC	BWC
Transverse load	597	400
Vertical load	591	500
Longitudinal load	-	1945

The D-shackle is made of high tensile steel. Assume permissible stress of high tensile steel as 2,500kg/cm^2 and 2,300kg/cm^2 in tension and bearing respectively.

Normal condition

Area of one leg = π / 4 x (1.6) = 2.01 sq.cm.

Assuming the total load to be the sum of vertical and transverse loads (conservative), the design load

= 597 + 591

= 1,188

Tensile stress = 1,188 / 2 x 1 / 2.01 = 295.5 kg/cm^2

Factor of safety = 2,500 / 295.5 = 8.46

Shearing stress in the bolt = $\dfrac{597}{\frac{\pi}{4} x\, 2^2}$

= 190 kg/cm^2

Factory of safety = 2,300 / 190 = 12.1

Broken-wire condition

Assuming the total load to be sum of the loads listed for broken-wire condition,

Tensile stress in shackle = 2,845 / 2 x 1 / 2.01

= 707.711 kg/cm^2

Factor of safety = 2,500 / 707.7 = 3.53

Shearing stress in the bolt = $\dfrac{2{,}845}{2\, x \frac{\pi}{4} x \left(2\right)^2}$

= 452.8

Factor of safety = 2,300 / 452.8 = 5.07

Hence safe.